T. F. Implícita:

$F : \mathbb{R}^n \to \mathbb{R}^m$, $m < n$, C^1, $\boxed{F(x) = 0}$

$$DF(a) = \Big[\underbrace{\begin{bmatrix} \cdot & \cdot & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \end{bmatrix}}_{m \times (n-m)} \underbrace{\begin{bmatrix} \cdot \\ \cdot \\ \cdot \end{bmatrix}}_{m \times m} \Big]_{m \times n}$$

$F(a) = 0$

$\det \neq 0$

$\Rightarrow$ Variáveis no bloco c/ $\det \neq 0$ são funções das restantes (de classe C^1).

$$\mathbb{R}^n = \mathbb{R}^{n-m} \times \mathbb{R}^m$$
$$(x \quad , \quad y)$$

$$\mathbb{R}^2 = \mathbb{R} \times \mathbb{R} = \{(x, y),\ x, y \in \mathbb{R}\}$$
$$(x_1, x_2)$$

$\mathbb{R}^3$: $(\underbrace{x_1}_{x}, \underbrace{x_2, x_3}_{y}) = (x, y)$

$$\mathbb{R}^3 = \mathbb{R} \times \mathbb{R}^2$$

ou

$(\underbrace{x_1, x_2}_{x}, \underbrace{x_3}_{y}) = (x, y)$

$$\mathbb{R}^3 = \mathbb{R}^2 \times \mathbb{R} \quad \text{etc.}$$

Geral:

m = nº de equações

n = nº de variáveis

$$\mathbb{R}^n = \mathbb{R}^{n-m} \times \mathbb{R}^m$$

$$F : \mathbb{R}^n \to \mathbb{R}^m, \quad m < n, \quad C^1$$

$$F(x, y) = 0$$

$$x = (x_1, x_2, \ldots, x_{n-m})$$

$$y = (x_{n-m+1}, \ldots, x_n)$$

$\mathbb{R}^3$: 1) $F(x,y,z)=0$

$F:\mathbb{R}^3 \to \mathbb{R}$,

$n=3$, $m=1$ $\quad \mathbb{R}^3 = \mathbb{R}^2 \times \mathbb{R}$ 1

$$DF(a,b,c) = \begin{bmatrix} \frac{\partial F}{\partial x} & \frac{\partial F}{\partial y} & \underbrace{\frac{\partial F}{\partial z}}_{\neq 0} \end{bmatrix}_{(a,b,c)}$$

$\Longrightarrow z = z(x,y)$

1 variável em função das 2 restantes.

2) $\begin{cases} F_1(x,y,z)=0 \\ F_2(x,y,z)=0 \end{cases}$ $\quad n=3$, $m=2$

$F=(F_1,F_2): \mathbb{R}^3 \to \mathbb{R}^2$

$$\mathbb{R}^3 = \mathbb{R}^{(1)} \times \mathbb{R}^{(2)}$$

$$DF(a,b,c) = \begin{bmatrix} \frac{\partial F_1}{\partial x} & \frac{\partial F_1}{\partial y} & \frac{\partial F_1}{\partial z} \\ \frac{\partial F_2}{\partial x} & \frac{\partial F_2}{\partial y} & \frac{\partial F_2}{\partial z} \end{bmatrix}_{(a,b,c)}$$

$\det \neq 0$

$$\Rightarrow \quad y = y(x) \quad e \quad z = z(x)$$

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Geral: $F : \mathbb{R}^n \to \mathbb{R}^m$, $m < n$, C^1

$$F(a,b,c) = 0$$

$$DF(a,b,c) = \left[D_x F \;\vdots\; \boxed{D_y F} \right]$$

$m \times m$

$m \times n$

$$\mathbb{R}^n = \mathbb{R}^{n-m} \times \mathbb{R}^m$$

$\det \neq 0$

$$\boxed{\det D_y F(a,b) \neq 0}$$

$$\mathbb{R}^n = \mathbb{R}^{n-m} \times \mathbb{R}^m$$

$$\uparrow \quad \uparrow$$

$$x \quad y$$

$$a \quad b$$

$$\Rightarrow \quad y = f\overset{b}{(x)}$$

$$f : \mathbb{R}^{n-m} \longrightarrow \mathbb{R}^m, \ C^1$$

T.F. Implícita

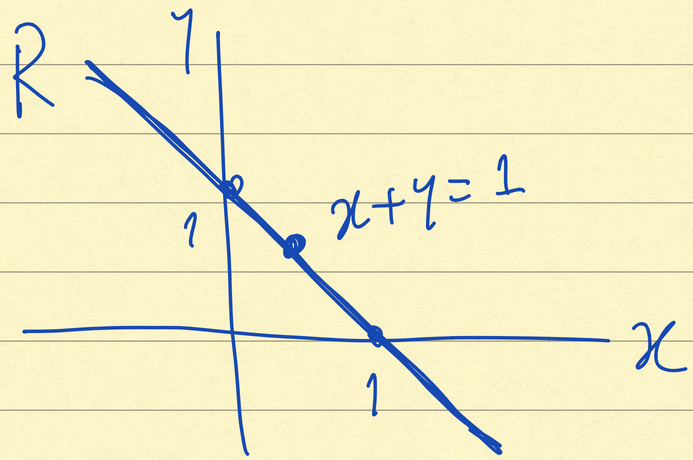
$$F(x,y) = 0 \ \underset{\boxed{\text{localmente}}}{\Leftrightarrow} \ y = f(x)$$

$$\underline{\qquad\qquad} \ // \ \underline{\qquad\qquad}$$

Conjuntos definidos por equações

Exemplos:

1) $R = \{(x,y) \in \mathbb{R}^2 : x + y = 1\}$



$$\underbrace{x+y-1}=0 \iff y=\underbrace{1-x} \iff x=\underbrace{1-y}$$

linear

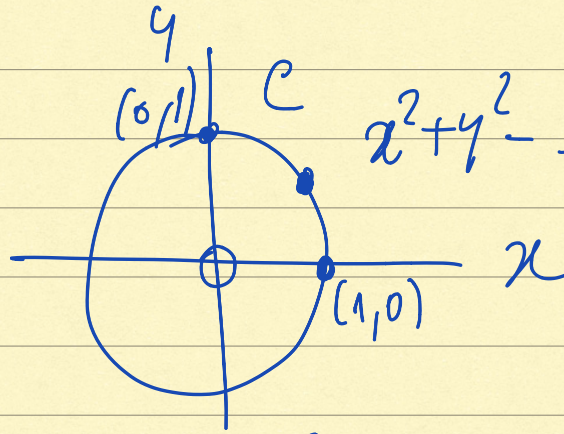
$$F(x,y)=0 \iff y=f(x) \iff x=h(y)$$

fazer as contas

$$DF(x,y)=\begin{bmatrix}1 & 1\end{bmatrix} \neq \begin{bmatrix}0 & 0\end{bmatrix} \quad \forall (x,y)\in R$$

2) $C=\{(x,y)\in\mathbb{R}^2 : x^2+y^2=1\}$

S^1



$x^2+y^2-1=0$

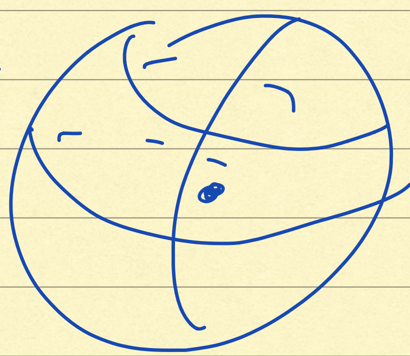
$$F(x,y) = x^2+y^2-1 = 0, \quad F:\mathbb{R}^2 \to \mathbb{R}, \ C^1$$

$$DF(x,y) = [2x \quad 2y] \neq [0 \quad 0]$$

$$\forall (x,y) \in C$$

3) $S^2 = \{(x,y,z) \in \mathbb{R}^3 : x^2+y^2+z^2 = 1\}$

esfera



$$F(x,y,z) = x^2+y^2+z^2-1 = 0$$

$$F:\mathbb{R}^3 \to \mathbb{R}, \ C^1$$

$$DF(x,y,z) = [2x \quad 2y \quad 2z] \neq [0 \quad 0 \quad 0]$$

$$\forall (x,y,z) \in S^2$$

4) $L = \{(x,y,z) \in \mathbb{R}^3 : x^2+y^2+z^2=1 \, ; \, x = y\}$

$$F(x,y,z) = \left(x^2+y^2+z^2-1, \; x-y\right) = (0,0)$$

$$F: \mathbb{R}^3 \longrightarrow \mathbb{R}^2, \quad C^1$$

$$DF(x,y,z) = \begin{bmatrix} 2x & 2y & 2z \\ 1 & -1 & 0 \end{bmatrix}$$

suponha-se que os três blocos quadrados têm determinante = 0:

$$\begin{cases} -2x-2y=0 \\ 2z=0 \\ -2z=0 \end{cases} \iff \begin{cases} 2x+2y=0 \\ z=0 \end{cases} \begin{cases} y=-x \\ z=0 \end{cases}$$

$$\begin{cases} y = -x \\ z = 0 \end{cases}$$ → Mas em L:

$$\begin{cases} x^2 + y^2 + z^2 = 1 \\ y = x \end{cases}$$

$$\begin{cases} x^2 + y^2 = 1 \\ y = -x \;;\; y = x \end{cases}$$

$$\begin{cases} 0 = 1 \\ x = y = 0 \end{cases}$$ absurdo

⇒ não pertencem a L.

⇒ O teorema da função implícita aplica-se em qualquer ponto de L.